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FIELD MEASUREMENT OFF THE MIDPLANE

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On several recent occasions, the need for measuring the field off the midplane of the synchrotron magnets has been brought up. The purpose of this note is to point out that, so far as characteristics of the particle orbits are concerned, measurement of the field on the midplane is all that is necessary, provided that the measurement is made with adequate precision.

It is easy to see that the field off the midplane is given in terms of the field on the midplane by Maxwell's equations. Take a right-handed coordinate system where the z-axis is along the closed orbit lying in the midplane (the xz plane) and the y-axis is perpendicular to the midplane. In expanded forms away from the midplane, the components of the magnetic field can be written as

$$\begin{cases} B_y = B_y^{(0)} + B_y^{(2)} \dfrac{y^2}{2!} + B_y^{(4)} \dfrac{y^4}{4!} + \dots \\ B_x = B_x^{(1)} y + B_x^{(3)} \dfrac{y^3}{3!} + \dots \\ B_z = B_z^{(1)} y + B_z^{(3)} \dfrac{y^3}{3!} + \dots, \end{cases}$$

where the coefficients $B_x^{(n)}$, $B_y^{(n)}$, and $B_z^{(n)}$ are functions of x and z. The curl $\vec{B} = 0$ equation,

$$\begin{cases} \dfrac{\partial B_x}{\partial y} = \dfrac{\partial B_y}{\partial x} \\[2ex] \dfrac{\partial B_y}{\partial z} = (1+kx)\dfrac{\partial B_z}{\partial y} \\[2ex] \dfrac{\partial B_x}{\partial z} = \dfrac{\partial}{\partial x}\Big[(1+kx)B_z\Big] , \end{cases}$$

where k is the curvature of the closed orbit (on the z-axis), gives immediately

$$\begin{cases} B_x{}^{(1)} = \dfrac{\partial B_y{}^{(0)}}{\partial x} \\[2ex] B_x{}^{(3)} = \dfrac{\partial B_y{}^{(2)}}{\partial x} \\[1ex] \text{etc.} \end{cases} \quad \text{and} \quad \begin{cases} B_z{}^{(1)} = \dfrac{1}{1+kx}\dfrac{\partial B_y{}^{(0)}}{\partial z} \\[2ex] B_z{}^{(3)} = \dfrac{1}{1+kx}\dfrac{\partial B_y{}^{(2)}}{\partial z} \\[1ex] \text{etc.} \end{cases} \tag{1}$$

The div $\vec{B} = 0$ equation,

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{1}{1+kx}\frac{\partial B_z}{\partial z} + \frac{k}{1}$$

gives

$$\begin{cases} B_y{}^{(2)} = -\dfrac{1}{1+kx}\left[k\dfrac{\partial B_y{}^{(0)}}{\partial x} + \dfrac{\partial}{\partial z}\left(\dfrac{1}{1+kx}\dfrac{\partial B_y{}^{(0)}}{\partial z}\right)\right] - \dfrac{\partial^2 B_y{}^{(0)}}{\partial x^2} \\[3ex] B_y{}^{(4)} = -\dfrac{1}{1+kx}\left[k\dfrac{\partial B_y{}^{(2)}}{\partial x} + \dfrac{\partial}{\partial z}\left(\dfrac{1}{1+kx}\dfrac{\partial B_y{}^{(2)}}{\partial z}\right)\right] - \dfrac{\partial^2 B_y{}^{(2)}}{\partial x^2} \\[2ex] \text{etc.} \end{cases} \tag{2}$$

Relations 1 and 2 express all the field coefficients in terms of $B_y^{(0)}$, the field on the midplane. These relationships have been incorporated in the mathematical treatment of the orbit properties together with such by-products as the error analysis. Hence, it is possible to specify the required field configuration, error tolerances, etc. only on the midplane.

This simple analysis can be easily generalized to the case where the fields are not symmetric about the midplane, that is, when $B_x^{(0)}$ and $B_z^{(0)}$ do not vanish.

It may be noted also that particular terms in the midplane and off-midplane fields are related.[1] For example, the sextupole term in the radial (x) equation ($1/2\ b_3x^2$ in the notation of Reference 1), which gives rise to the third-integral resonances of the form $3\nu_x = m$ (m integral), also appears in the vertical (y) equation as b_3xy, giving rise to the coupling resonances $\nu_x \pm 2\nu_y = m$.

It is, therefore, necessary only to measure the field on the midplane. As long as the midplane field is as specified within tolerance, the orbit characteristics will be acceptable no matter how horrible the off-midplane field may look.

REFERENCE

[1] L. C. Teng, Formulas for Resonances of Transverse Oscillations in a Circular Accelerator, FN-183, March 27, 1969.